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ONE PROBLEM OF THE THEORY OF DIMENSIONAL ELECTROCHEMICAL MACHINING OF METALS

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A method is proposed for determining the shape of the anode-article boundary for a given shape of the cathode-tool in plane problems of the theory of dimensional electrochemical machining of metals. Under the assumptions used, the boundary of the anode-article is divided into the working zone, where metal dissolution occurs, and an adjacent zone, where the treatment (dissolution) is terminated. The initial problem is reduced to a problem of a fictitious plane-parallel potential flow of an ideal fluid with a nonlinear condition on the free surface. The point of separation of the fictitious flow from the solid boundary corresponds to the point separating these two zones of the anode boundary. The Brillouin–Will condition of smooth separation is imposed at the separation point to construct a closed system of equations determining the problem solution.

Key words: *electrochemical machining of metals, hydrodynamic analogy, numerical-analytical solution.*

Introduction. Dimensional electrochemical machining (ECM) is one of the modern methods of fabricating articles with a prescribed shape, size, and surface quality from metals and alloys by means of anodic dissolution of the workpiece in a flow-through electrolyte with the use of a special cathode-tool. A detailed description of the process and technology of dimensional ECM can be found in [1–4].

To ensure high accuracy of copying the shape and size of the cathode-tool on the machined workpiece with a prescribed allowance for processing, it is necessary to localize the process of electrochemical dissolution of the metal in the zone allocated for machining. The dissolution process outside this zone should be drastically decelerated down to its complete termination. Based on the analysis of electrode processes for various electrolytes, Davydov and Kozak [4] showed that localization of metal dissolution substantially depends on the electrolyte composition, metal properties, and conditions of the process. For low values of the current density i, the current efficiency η (fraction of the total amount of electricity spent on anodic dissolution of the metal or alloy) for reactions of anodic dissolution of the metal [3] in $NaNO_3$ and $NaClO_3$ solutions is actually equal to zero. The value of η starts to increase with increasing i as a result of anodic activation of the metal [4] under the action of anions of these salts at a certain critical current density $i_{\rm cr}$. Metal dissolution is mainly concentrated on machined workpiece areas with the smallest distance between the electrodes and with the maximum dissolution rate. The use of $NaNO_3$ and $NaClO_3$ solutions ensures high accuracy of workpiece machining.

Certain necessary conditions being satisfied, the surface after long-time machining acquires a particular shape further unchanged with time, which is called the stationary shape [4]. In the stationary regime, the shape of the machined surface in a cathode-fitted moving coordinate system remains unchanged, i.e., the anode surface moves together with the cathode with a constant velocity V_c.

A numerical-analytical solution of a plane problem of the ECM theory is constructed in the present paper within the framework of a model of an ideal process [4]. The stationary shape of the surface of an article machined with a two-sided cathode-tool that has an insulated side face is determined with allowance for the localization properties of the above-mentioned electrolytes.

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1. Model of the Process. In the model of an ideal ECM process, the electric field can be described by the Laplace equation

$$\nabla^2 u = 0, \tag{1.1}$$

where u is the electric field potential. The potential $u_a = U - E_a$ on the article (anode) surface is determined as the difference between the prescribed unchanged value of the voltage U between the electrodes and the jump of the electrode potential E_a of the anode. The potential on the working surface of the cathode-tool (cathode) $u_c = -E_c$ is the jump of the electrode potential of the cathode. The model of an ideal process implies that E_a and E_c are constant jumps of the potentials averaged over the electrode surfaces [4].

The distribution of the current density i on the stationary anode boundary is determined by the equality [4]

$$\eta(i_a) i_a = (\rho V_c/\varepsilon)\cos\theta, \tag{1.2}$$

where i_a is the anode current density, ε is the electrochemical equivalent of the metal, ρ is the density of the anode material, and θ is the angle between the vector $\boldsymbol{V}_c$ of velocity of cathode motion and the vector $\boldsymbol{n}_a$ of the external normal at this point of the anode boundary. Condition (1.2) takes into account that η is a function of i.

Sedykin et al. [5] plotted the current efficiency as a function of the current density for the case of treatment of the 5KhNM steel in $NaNO_3$ and $NaClO_3$ solutions with different concentrations C. For these electrolytes, the analytical dependence $\eta(i_a)$ can be presented as follows [6]:

$$\eta(i_a) = \begin{cases} 0, & i_a \leqslant i_{\rm cr}, \\ a_0 + a_1/i_a, & i_a > i_{\rm cr} \end{cases} \tag{1.3}$$

(a_0, a_1, and $i_{\rm cr}$ are constants).

Using Eqs. (1.2) and (1.3) and the equality $i_a = æ\,\partial u/\partial n_a$, where æ is the specific electrical conductivity of the medium, we obtain the boundary condition on the anode boundary:

$$æ\,\frac{\partial u}{\partial n_a} = -\frac{a_1}{a_0} + \frac{\rho V_c}{a_0 \varepsilon}\cos\theta. \tag{1.4}$$

Let us now pass to dimensionless variables $\psi = (u - u_c)/(u_a - u_c)$ and $n = n_a/H$ [$H = æ(u_a - u_c)/i_0$ is the characteristic size and $i_0 = \rho V_c/\varepsilon$ is the characteristic current density] [7]. It follows from Eq. (1.1) that the function ψ satisfies the Laplace equation $\nabla^2\psi = 0$ in the gap between the electrodes. The following conditions are satisfied on the electrode boundaries:

$$\psi_a = 1, \qquad \psi_c = 0. \tag{1.5}$$

Condition (1.4) takes the form

$$\frac{\partial \psi}{\partial n} = a + b\cos\theta, \qquad a = -\frac{a_1}{a_0 i_0}, \quad b = \frac{1}{a_0}. \tag{1.6}$$

The following condition is satisfied on the lines of symmetry and on the boundaries of the dielectric coatings:

$$\frac{\partial \psi}{\partial n} = 0. \tag{1.7}$$

Klokov et al. [8] revealed an important hydrodynamic analogy of the problem of the theory of dimensional ECM of metals for the stationary regime and demonstrated that the theory of boundary-value problems for analytical functions can be used to study plane-parallel problems of the theory of electrochemical forming.

According to the hydrodynamic analogy, the plane potential electric field is modeled by a fictitious plane-parallel potential flow of an ideal incompressible fluid. We assume that the moving coordinate system is fitted to the cathode and the direction of the ordinate axis coincides with the direction of cathode motion with respect to the motionless anode. If we introduce a complex potential $W(x,y) = \varphi(x,y) + i\psi(x,y)$ of the electrostatic field [the function $\varphi(x,y)$ corresponds to the force function of the electrostatic field], then the equality $\partial\psi/\partial n = V$ is valid along the line $\psi = \text{const}$. Then, in the hydrodynamic interpretation of dimensional ECM problems, the vector $\boldsymbol{V}$ corresponds to the vector of velocity of this fictitious flow. The angle of inclination of the vector $\boldsymbol{V}$ to the abscissa axis at this point of the anode boundary coincides with accuracy to its sign with the angle between the direction of cathode motion and the vector of the external normal at the same point. Condition (1.6) yields

$$V = a + b\cos\theta. \tag{1.8}$$

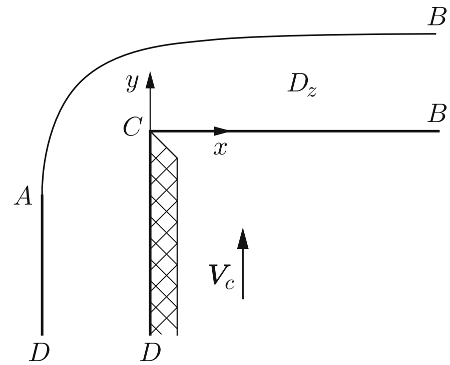

Fig. 1. Sketch of the interelectrode gap: BCD is the cathode boundary, BC is the working surface of the cathode, which is perpendicular to the direction of cathode motion, and CD is the boundary of the dielectric coating.

Here V and θ are the absolute value and the argument of the velocity vector, respectively. Equation (1.8) determines the hodograph of velocity of this flow on the unknown anode boundary. The hydrodynamic analogy facilitates the formulation of boundary-value problems, which allows the methods used in the theory of ideal fluid jets [9, 10] to be used for studying problems of dimensional ECM of metals.

2. Formulation of the Problem and Its Numerical-Analytical Solution. A sketch of the interelectrode gap is shown in Fig. 1. The electrode-tool shown here can be used for forming cavities in machine articles, processing of edges, and other technological processes.

According to condition (1.3), the sought anode boundary can be divided into two zones. Metal dissolution occurs in the zone AB. The normal derivative $\partial\psi/\partial n$ on this segment satisfies condition (1.6). In the region modeled by the vertical straight-line segment AD, the anodic current efficiency is almost zero, and no metal dissolution occurs. The current density on the segment AD changes from $i_{\rm cr}$ at the point A to zero at an infinitely distant point D. The location of the point A is not known in advance and has to be found in the course of solving the problem. The vector $\boldsymbol{V}_c$ indicates the direction of cathode motion. The abscissa axis is orthogonal to the direction of cathode motion.

The hydrodynamic analog is the problem of the theory of plane stationary flows of an ideal incompressible fluid on determining the boundary of the free surface with a prescribed law of velocity variation (1.8). The flow is generated by a system of sources continuously distributed along the line CD. Further we consider a jet problem corresponding to the formulation described above.

In accordance with the *a priori* concepts about the shape of the free boundary AB on this segment, the angle θ monotonically decreases from $\pi/2$ to zero, i.e., there are no inflection points on the free boundary. Along the boundary AD, the velocity should monotonically decrease from a constant value $V = a$ at the point A to zero at infinity (point D). For these requirements to be satisfied, we use the Brillouin–Will condition of smooth separation, which is known in hydrodynamics [9, 10]. According to this condition, the curvature of the free surface AB at the point A is finite and coincides with the curvature of the wall AD, i.e., it is equal to zero in the problem considered.

We introduce an auxiliary complex variable $t = \xi + i\eta$ changing in the domain $D_t = \{|t| \leqslant 1, \eta \geqslant 0\}$ (Fig. 2) and seek for a function $z(t)$ that ensures conformal mapping of a half-circle of unit radius onto the flow region. Correspondence of the points indicated in Figs. 1 and 2 is required. Instead of the function $z(t)$, it is possible to seek for the Joukowski function [10]

$$\chi(t) = \ln\Big(\frac{1}{V_0}\,\frac{dW}{dz}\Big) = r - i\theta, \qquad r = \ln\Big(\frac{V}{V_0}\Big),$$

where $V_0 = a + b$ is the velocity of the fictitious flow at the point B ($t = 1$). The function $\chi(t)$ is related to the functions $W(t)$ and $z(t)$ as

$$\frac{dz(t)}{dt} = \frac{\exp\left(-\chi(t)\right)}{V_0}\,\frac{dW}{dt}. \tag{2.1}$$

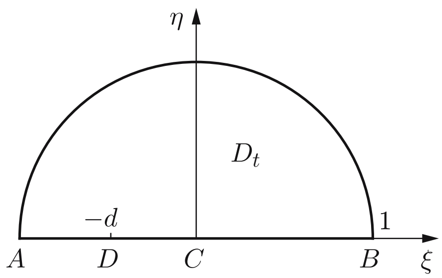

Fig. 2

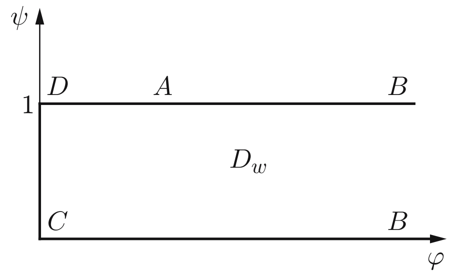

Fig. 3

Fig. 2. Plane of the parametric variable t.

Fig. 3. Domain of variation of the complex potential $W(t)$.

According to conditions (1.5) and (1.7), the domain D_w of variation of the complex potential is a half-band of unit width (Fig. 3). The expression for the derivative of the complex potential has the form

$$\frac{dW}{dt} = \frac{1+d}{\pi}\,\frac{t+1}{(1-t)\sqrt{t(t+d)(1+td)}}.$$

We present the function $\chi(t)$ as the sum [10]

$$\chi(t) = \chi_*(t) + \omega(t), \tag{2.2}$$

where $\omega(t)$ is the function, which is analytical in the domain of variation of the variable t; the function $\chi_*(t) = r_* - i\theta_*$ corresponds to the flow in accordance with the prescribed scheme (see Fig. 1) with the condition $V_* = V_0$ on the anode boundary AB; $r_* = \ln(V_*/V_0)$. It follows from condition (1.8) and the flow scheme (see Fig. 1) that the functions $\chi(t)$ and $\chi_*(t)$ on the boundary of the domain D_t satisfy the following conditions:

$$a + b\cos\theta(t) - V_0\exp(r(t)) = 0, \qquad t = \exp(i\sigma), \quad \sigma \in [0,\pi], \quad r(1) = 0,$$

$$\begin{gathered}\operatorname{Im}\chi(\xi) = \operatorname{Im}\chi_*(\xi) = -\pi/2, \qquad \xi \in [-1,-d),\\ \operatorname{Im}\chi(\xi) = \operatorname{Im}\chi_*(\xi) = -\pi, \quad \xi \in (-d,0),\\ \operatorname{Im}\chi(\xi) = \operatorname{Im}\chi_*(\xi) = 0, \qquad \xi \in (0,1],\\ \operatorname{Re}\chi_*(t) = 0, \qquad t = \exp(i\sigma), \quad \sigma \in [0,\pi].\end{gathered} \tag{2.3}$$

Using the method of Chaplygin's singular points [10], we

$$\chi_*(t) = -\ln(t) + \frac{1}{2}\ln\Big(\frac{t+d}{1+td}\Big), \tag{2.4}$$

where d is the coordinate of the image of the point D in the domain D_t.

Taking into account equality (2.2) and boundary conditions (2.3), we obtain the following nonlinear boundary-value problem for the function $\omega(t)$:

$$a + b\cos(T+\mu) - V_0\exp(\lambda) = 0; \tag{2.5}$$

$$\operatorname{Im}\omega(\xi) = 0, \quad \xi \in [-1,1], \qquad \operatorname{Re}\omega(1) = 0. \tag{2.6}$$

Here

$$T = \operatorname{Im}\chi_*(\exp(i\sigma)), \qquad \mu = \operatorname{Im}\omega(\exp(i\sigma)), \qquad \lambda = \operatorname{Re}\omega(\exp(i\sigma)).$$

By virtue of condition (2.6), the function $\omega(t)$, which is the solution of the boundary-value problem (2.5), (2.6), has an expansion of the form

$$\omega(t) = \sum_{k=0}^{\infty} c_k t^k, \qquad c_0 = -\sum_{k=1}^{\infty} c_k, \tag{2.7}$$

where c_k are real constants.

TABLE 1

Calculated Values of the Parameter d and Coordinates of the Point A

N	d	x_A	y_A
60	0.82140029053845	−0.660722441393243	−0.707434137085583
120	0.82137235096538	−0.660810055242296	−0.707194346531225
240	0.82136539305651	−0.660809957626806	−0.707175375037151

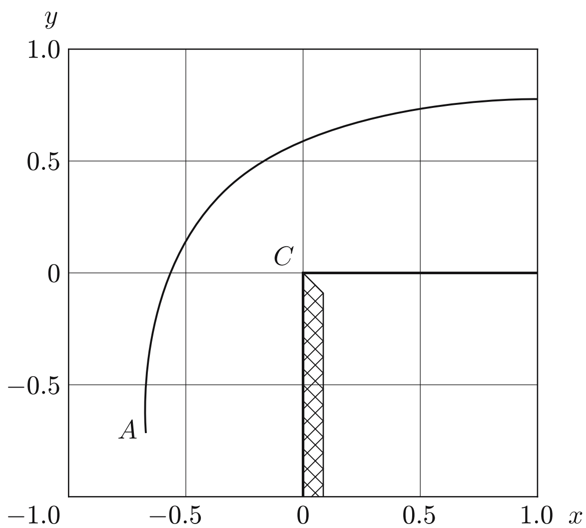

Fig. 4

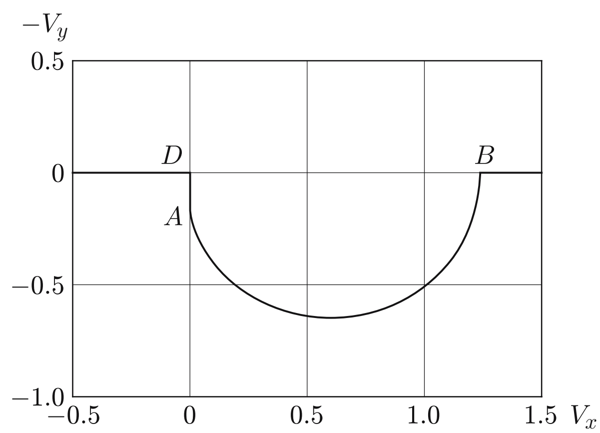

Fig. 5

Fig. 4. Results of calculation of the anode boundary (the hatched region shows the cathode-tool).

Fig. 5. Domain of the velocity hodograph.

According to [9, 10], the condition of smooth separation at the point A can be presented as the equality

$$\frac{d\theta}{d\sigma} = 0 \quad \text{at} \quad \sigma = \pi. \tag{2.8}$$

With the use of Eqs. (2.2), (2.4), and (2.7), this relation is transformed to

$$1 - \frac{1+d}{2(1-d)} - \sum_{k=1}^{\infty} c_k k(-1)^k = 0.$$

It follows from here that

$$d = \frac{F-1}{F+1}, \qquad F = 2\Big(1 - \sum_{k=1}^{\infty} c_k k(-1)^k\Big). \tag{2.9}$$

All necessary geometric characteristics of the flow can be found from the parametric dependence (2.1):

$$dz = \frac{1+d}{\pi V_0 \exp(c_0)} \frac{1+t}{1-t} \frac{\sqrt{t}}{t+d} \exp\Big(-\sum_{n=1}^{\infty} c_n t^n\Big) dt.$$

To solve the problem numerically, we define the coefficients a_0 and a_1 characterizing the properties of the electrolyte and the characteristic current density i_0. The coefficients of expansion (2.7) are defined so that condition (2.5) is satisfied on the sought anode boundary. The problem is solved by the method of collocation, which is widely used in solving problems of hydrodynamics of plane stationary flows of an ideal fluid with free surfaces [10]. The system of equations for calculating the coefficients of expansion (2.7) is solved by the Newton method together with Eq. (2.9) derived for determining the mathematical parameter d.

3. Calculation Results. To estimate the accuracy of numerical results depending on the number of collocation points N, we performed test calculations with the following values of parameters: $i_0 = 100$ A/cm^2, $a_0 = 0.906$, and $a_1 = -12.818$ [the values of a_0 and a_1 refer to the $NaNO_3$ solution ($C = 15\%$)] [6]. The values of the mathematical parameter d and the coordinates of the point A for different values of the number N are listed in Table 1. At $N = 120$, the approximate solution can be found in this case with accuracy to 10^{-4}. Figure 4 shows the results of calculating the anode boundary. Figure 5 shows the domain of variation of the function

$$\frac{dW}{dz} = V_0 \exp(\chi(t)) = V \exp(-i\theta) = V_x - iV_y$$

for the value of the parameter $d_* \approx 0.821365$ found from condition (2.8) of smooth separation. The absolute value of velocity along the free boundary AB monotonically decreases from $a + b \approx 1.245$ at the point B to $a \approx 0.141$ at the point A. The velocity along the boundary AD monotonically decreases, reaching zero at infinity (point D).

At $d \neq d_*$, the condition of monotonic variation of velocity along the free boundary AB and the boundary AD is violated, which does not allow a free boundary satisfying the boundary condition (1.8) to be constructed.

Conclusions. The problem of calculating the shape of the stationary anode boundary for a given configuration of the cathode-tool is solved in the paper on the basis of a two-dimensional mathematical model of an ideal process [4] with allowance for a particular dependence of the current efficiency on the anode current density. The use of the smooth separation condition [9] allows us to determine the free boundary satisfying the boundary condition (1.8). The procedure ensures the only possible value of the gap between the insulated side face of the cathode and the vertical segment of the machined surface.

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